**Homework Assignment 4 – Solutions to 7 to be added**

Grading based on 1,2,3 (and bouns)

1. Charles likes to drink sparkling waters. His preferences on San Pelligrino water (x) and Vintage seltzer water (y) can be represented by the utility function U(x,y) = 4**ln**x + **ln**y where both are measured in liters.

1. If the price of San Pelligrino is Px per liter, the price of Vintage is Py per liter and he has $I to spend each month on sparkling waters then find his demand functions for San Pelligrino and Vintage.



1. Suppose that Px = $1.50, Py = $.50 and I = $60. Using your answer to (a) find his demands for San Pelligrino and Vintage. Illustrate your answer in an indifference curve diagram. Include in your diagram an indifference curve through his best bundle.







The domestic sparkling water industry would like to boost the sales of a domestic sparkling waters. As a consequence the industry (which includes the makers of Vintage) lobbies congress to place a tariff on the imports of foreign sparkling water (including San Pelligrino). The effect of the tariff is to raise the price of San Pelligrino to $2 per liter.

1. After the price increase what are Charles’s new demands for San Pelligrino and Vintage? Illustrate the effect of the subsidy in your diagram for part (b). In your illustration be sure to clearly label the substitution and income effects.



1. Did the tariff on San Pelligrino achieve the ends of domestic sparkling water producers to boost thedemand for domestic sparkling water? Use your substitution and income effects to explain your answer.



2 Edith's preferences over donuts (x) and croissants (y) can be represented by the utility function u(x,y) = min[2x,y].

1. Find Edith's Marshallian demand functions for donuts and croissants.

1) Tangency

2) Feasibility

1. If Px = 2, Py = 3 and I = 80 then how many donuts and croissants does she buy? Illustrate your answer in the diagram below.

Icc - part k

1. If income rises to 88, then what will be her new best bundle? Illustrate the new budget line and best bundle in your diagram.
2. Calculate the income and the own price elasticity of Edith's demand for donuts. Write your answer as functions of prices and income only.

hence:

In class we derived the equality where is the share of income spent on the x good and ηx is the income elasticity of the x good, donuts (similarly θy and ηy are the share of income and the income elasticity of the y good, croissants.

1. Derive the expression .

From the budget constraint , take derivatives on both sides to get:

Rearrange to get the formula.

1. Use your income elasticities from part (c) to verify that this expression holds for Edith's demands.

From , and same for x, we have

1. In general (not specifically for Edith's demands) how does θx vary when I changes (i.e. find )?

and so

1. If ηx < 1 then what is the sign of the derivative in part (f)? If ηx = 1 then what is the sign of the derivative in part (f)? If a good is a necessity then as income rises does the share of income spent on that good rise or fall? There is additional room on the next page.

Rearranging the expression above we get

So if the expression is negative, and if the expression is zero. Thus, for a necessity, the share of income spent on the good declines or stays constant as income grows.

1. Given the income elasticity that you found for donuts in part (d) what will be the derivative of θx with respect to I for Edith's demands?

We have which implies =0

1. Write θx as a function of only prices and income for Edith's demand for donuts. Verify your answer to part i (ie calculate the derivative directly and show that it equals your answer to part (i)?

Since we have =0

1. The income-consumption curve (icc) shows how Edith's consumption of donuts and croissants increases as her income increases. Illustrate the icc in your diagram above.

(diagram).

3. Bart purchases food (x) and lottery tickets (y). His preferences on units of food (x) and lottery tickets (y) can be represented by the utility function U(x,y) = min[2x,y].

1. If the price of a unit of food is Px, the price of a lottery ticket is Py and he has $I to spend then find his demand functions for food and lottery tickets.

We want to solve the following optimization problem:



Note that the tangency condition does not involve the MRS and price ratios because the preferences are given by a perfect complements function. The first order conditions are:

“Tangency”: 

Feasibility: 

Substitute the tangency condition into the feasibility condition:



Now that we have x we can get y from:



To summarize our results, the demand curves are:



1. Suppose that Px = $2, Py = $1 and I = $24. Using your answer to (a) find his demands for food and lottery tickets. Illustrate your answer in an indifference curve diagram on the next page.



Lottery Tickets

Units of food

24

12

24

6

12

8

16

A=B

C

A = original point

B = substitution point (same as A here)

C = final point

A 🡪 B substitution effect (0 in this case)

B 🡪 C income effect

IE

IE

The government would like to encourage Bart to eat more but not to buy more lottery tickets As a consequence congress decides to subsidize the purchase of food by $1 per unit. Thus the price of a unit of food falls to $1.

1. After the price decrease what are Bart’s new demands for food and lottery tickets? Illustrate the effect of the subsidy in your diagram for part (b). In your illustration be sure to clearly label the substitution and income effects.



1. Did the subsidy achieve the goal of only increasing Bart’s consumption of food? Use your substitution and income effects to explain your answer.

No, the subsidy did not achieve its goal because the consumption of lottery tickets also increased. Because Bart’s preferences are given by a perfect complements function then there is no substitution effect. Thus the only effect is the income effect. With the price of food reduced, consumers are now relatively more wealthy, and since both goods are normal (as is evident in the demand functions), the income effect makes them purchase more of both goods

1. Using your answer to part (a), show that the cross price elasticity of lottery tickets with respect to the price of food is negative.

The cross price elasticity of lottery tickets with respect to the price of food is:



We know that and thus 

Substitute these results back to the expression for elasticity:



Since the prices are positive, the elasticity has to be negative.

1. Does a negative cross price elasticity imply that the substitution effect is negative? Briefly explain why or why not.

No it does not. Observe for example that in this problem the cross price elasticity is negative and the substitution effect is zero (i.e. not negative). The explanation is that the cross price elasticity includes both the substitution and the income effect. If the income effect is large enough, it can dominate the total effect and we can have a situation like the one shown in this problem where the cross-price elasticity is negative but the substitution effect is not.

1. Show directly that the substitution effect in this case is 0 by calculating the derivative of the Hicksian demand for lottery tickets with respect to the price of food.

We need to solve the expenditure minimization problem:



Tangency: 

Feasibility: 

The solution to this problem is:

 or:



None of these Hicksian demand functions has a price term in it, so their derivative w.r.t Px is zero.

4. In the state of New Francia a typical worker earns a wage of $10 per hour. She can work up to 20 hours per week tax free that is she can earn $200 per week before she must pay taxes on her earnings. For every dollar that she earns above 200 per week she must pay a 10% tax on her earnings. Hence her after tax wages are only $9 for all hours above 20 hours per week. The typical worker in New Francia has no outside income (no income other than wage income) and she uses her income to purchase a consumption good priced at $1 per unit. There are 168 hours in a week.

1. Illustrate the budget set of a typical worker in New Francia.

BLUE line

1. What is the opportunity cost of leisure if she is working 10 hours per week? What is the opportunity cost of leisure if she is working 30 hours per week?

If she works 10 per week $10. If she works 30 hours per week $9

The typical worker in New Francia works 45 hours per week.

1. In your diagram in part (a) illustrate the indifference curve for the typical worker through her best bundle (at 45 hours per week of labor). In your diagram assume that leisure and the consumption good are not perfect complements.

IC : ORANGE

BC : GREEN

1. What is the (approximate) average hourly wage of the typical worker who is working 45 hours per week? What is the average tax rate?

Total earning= earnings before tax - tax= 10\*45-1\*25=425

Average tax rate = tax payment / earnings before tax=25/450=0.056 or 5.6%

New Francia is considering a “flat tax” system. Specifically the new tax system will charge the same tax rate on every single dollar earned so the typical worker will have to pay taxes on every dollar earned (including the first dollar earned). However, New Francia will lower the tax rate to the average tax rate that the typical worker was paying under the old tax system (the dollar figure you found in part (d)).

1. In your diagram in part (a) illustrate the new budget line associated with the new flat tax.

RED line

1. Will the typical worker still work 45 hours per week under the new contract? Briefly explain your answer.

No she will not work 45 hours per week since MRS at 45 hours is smaller than NEW slope of the budget line that pass through the OLD optimal point.

5. Barry must decide how to allocate his 24-hour day between non-wage activities, *leisure*, and wage activities, *labor*. For every hour of labor that he supplies he is paid by his employer $w. Barry can purchase bottles of wine (y) at a price of **$**10 per bottle (and wine is the only consumption good). Barry has non-wage income of $I and so his total income is equal to the sum of his wage income and his non-wage income. Finally assume that Barry’s preferences over leisure and wine can be represented by the following utility function: U(l,y) = ln(l) + ln(y)

1. If Barry consumes 24 hours of leisure then how many bottles of wine can he purchase? If Barry works 24 hours then how many bottles of wine can he purchase. Illustrate his budget set in a diagram below and write the equation for his budget line.

If Barry consumes 24 hours of leisure he can consume bottles of wine.

If Barry works 24 he can purchase bottles.

The slope of the budget line is given by the (real) wage

1. Find Barry’s demand for leisure as a function of w and I. What is Barry’s supply of labor as a function of w and I?

And the labor supply is

For parts (c) through (e) below set $I = $40.

1. Given your answer to (b) above what will be Barry's supply curve of labor? Illustrate it below.

See the other exercise on labor supply.

1. Suppose that Barry’s wage is **$20**. Given the demand functions that you found above what are Barry’s demands for leisure and wine at this wage? What is his supply of labor? Illustrate your answer in your diagram in part (a). Be sure to include an indifference curve through his best bundle.
2. Suppose that Barry’s wage rises. Illustrate in your indifference curve diagram the effect of the increase in wages on his optimal choice of leisure and wine. Make sure to clearly indicate the income and substitution effects in your diagram. Given the slope of the labor supply curve which effect (the income or substitution) is larger on his demand for leisure?

See the graph for the other exercise on labor supply. The labor supply increases (you can take the derivative to check).

1. Hold wage constant at $20 per hour. Suppose that his non-wage income were to increase. Given your labor supply function from part (b) what will be the effect of an increase in non-wage income on his labor supply. Why did a change in non-wage income have a different effect on labor supply than the change in wage that you analyzed in part (e)?

A change in non-wage income increases leisure demand by:

And hence reduces labor supply. In this case, there is no substitution effect. The income effect leads the worker to work less.

6. There are 168 hours in a week. Assume that Betty earns $10 per hour and has no other source of income. She can use her income to purchase food priced at $1 per unit.

1. Illustrate Betty’s budget set.



The government would like to subsidize Betty’s wages. For the first 20 hours that she works in a week the government will pay her an additional $1 per hour (thus raising her wage to $11 for each hour worked up to 20). However, starting at 20 hours of work, the government will no longer subsidize her wages and in fact will start taxing them. In particular the government will levy a tax of $1 per hour on all hours worked above 20 hours (thus lowering her wages to $9 for every hour of work over 20).

1. In your diagram for part (a) illustrate the effect of the subsidy/tax program on the Betty’s budget set. In the diagram label the intercepts of the new budget line.
2. Find the bundle that is common to both budget lines.

**There are two bundles, (168,0) and (128,400). At the first bundle she is consuming only**

**leisure (all 168 hours of leisure). At the second bundle she is working 40 hours. At 40**

**hours of work her total subsidy is $20 and her total tax is $20. Thus she nets 0 from the**

**government and her income is the same before and after the program**

In parts (d)- (f) below you will examine 3 different best bundles on the after subsidy/tax budget line. For each of them you will need a separate diagram.

1. Suppose that after the subsidy/tax plan is implemented Betty chose to work 30 hours per week. Show that Betty is a net recipient of government funds (i.e. her total subsidy – total taxes > 0). Illustrate her choice in your diagram from part (a). In your diagram indicate the net income that she receives from the government.

**At 30 hours of work her total subsidy is $20 ($1 on each of the first 20 hours of work)**

**and her total tax is $10 ($1 on each of the 10 hours of work above 10). Thus her net subsidy is $10.**

1. Suppose that after the implementation of the subsidy/tax plan Betty chooses the bundle such that she nets 0 from the government’s program (total subsidy = total taxes). Briefly explain why Betty will be worse off under the new subsidy/tax program than she was before the program. You might find it useful to use a diagram in your explanation (in particular first locate the bundle such that she nets 0 from the program).



**If the best bundle on the Subsidy/Tax line is the bundle where her net receipts are 0**

**then we know that she is at the bundle (128,400) – the intersection of the two budget**

**lines. We know that the real wage associated with the Subsidy/Tax line at (128,400) is 9.**

**Thus her MRS at (128,400) must be 9. We know that the original budget line had a real**

**wage of 10. Thus her MRS at (128,400) is not equal to the real wage on the original**

**budget line (and it is on the original budget line) so we can conclude that it is NOT the**

**best bundle. In other words there are more preferred bundles on the original budget**

**line. Those bundles are indicated in the diagram above. Note that all of them have a**

**lower number of leisure hours (and a higher number of hours worked).**

1. Finally suppose Betty is indifferent between the two programs (so she is indifferent between her best bundle when there is no program and her best bundle when there is a program). Show that Betty will work less and be a net recipient of government income in this case. Again a diagram may be helpful.



**If she is indifferent between no program and the subsidy/ tax program then her best**

**bundle on each budget line must be on the same indifference curve as illustrated above.**

**We know that the best bundle on the Subsidy/Tax line must be to the right of 128 hours.**

**This follows from the observation that if the best bundle is to the left of (128.400) on the**

**Subsidy/Tax line then she would be strictly better off without the program (these**

**bundles are inside the budget set without the program). Hence the best bundle on the**

**Subsidy/Tax line must lie to the right of (128,400) (on the green portion above).**

**Similarly it must be the case that the best bundle on the original budget line must be to**

**the left of (128,400) because the bundles to the right of (128,400) on the original budget**

**line lie inside the budget set with the program. Again the green portion of the original**

**budget line represents the possible location of the best bundle on the original budget**

**line.**

**Hence the number of hours worked without the program must be higher than the**

**number of hours worked with the program. Because she is consuming more than 128**

**hours of leisure we know that she is a net recipient of government income.**

**Finally note that the best bundle on the Subsidy/Tax line cannot be on the portion of the**

**line above 148 hours of leisure. The best bundle on the original budget line must have**

**an MRS of 10. By diminishing MRS we know that the MRS must decline as the number**

**of hours increases. Since the Subsidy?tax line has a slope of 11 after 148 we know that**

**the best bundle on the subsidy/tax line cannot be on this portion of the budget line (it**

**cannot have an MRS of 11 and be on the same indifference curve.**

7. On homework 2 you found the demand functions for donuts (x) and bagels (y) and the utility function U(x,y) = x + 2 ln y where x represents the number of donuts and y represents the number of bagels. You also found the best bundle when prices and income were Px=1, Py=1, and I = 12 and when the price of x rose to 1.5.

1. Redraw your diagram illustrating the change in the best bundle due to the price change. On your new diagram illustrate the substitution and income effects of the price change.
2. Calculate the Hicksian demand functions for this utility function.
3. In your diagram for part (a), indicate the Hicksian demands for both goods at the original prices and original utility level and at the new prices and original utility level.

(H:Post solution for 2logx so that they have another example).

***Bonus****:* On homework assignment 2 you calculated the uncompensated demand curves for the utility function U(x,y) = lnx + 3lny.

1. For this utility function calculate the compensated demand curves for x and y.
2. Using your uncompensated and compensated demand curves calculate the partial derivative of both uncompensated and compensated demand for x with respect to px and the partial derivative of uncompensated demand with respect to income.
3. Using your answers for (b) verify the Slutsky equation.

*For the solution, the utility function is*  U(x,y) = 2lnx + 3lny.

1. For this utility function calculate the compensated demand curves for x and y.
2. Using your uncompensated and compensated demand curves calculate the partial derivative of both uncompensated and compensated demand for x with respect to px and the partial derivative of uncompensated demand with respect to income.
3. Using your answers for (b) verify the slutsky equation.



In words, find the bundle that achieves a utility level u at the minimum possible cost (note that u is a constant here).

The tangency condition still holds:



Substitute this back into the constraint:



Not that we have x, y is given by:



To summarize, the Hicksian demand functions are:



And the Marshallian demand functions (derived in last homework) are:



There are two Slutsky equations for good x:

 (1)

 (2)

We will show that the first one holds. The second one can be shown to hold true in a similar way:



The RHS does not look quite like the LHS yet. Note, however, that the constant u that we used for the cost minimization problem has to be equal to the utility achieved at the optimal point of the utility maximization problem. In other words:



The Slutsky equation holds ☺